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6. AUTHORS
Abolghassem Miamee, Andrzej Makagon

7. PERFORMING ORGANIZATION NAMES AND ADDRESSES
Hampton University
100 E. Queen Street

Hampton, VA 23668 -0108

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14. ABSTRACT
The research dealt with the prediction problem for periodically correlated sequences, that is the stochastic sequences whose mean and covariance structure vary with time in a periodic way. We aimed at sequences with large periods. It has been known already for years that in order to do a reliable forecasting of periodically correlated sequences with large period (or continuous time processes) the standard method of rephrasing the problems in terms of multivariate stationary sequences does not work because of a huge number of unknown parameters. Our

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UU	UU	UU	UU		Abolghassem Miamee
					19b. TELEPHONE NUMBER 757-727-5842

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Report Title

Final Report: Prediction Theory of Periodically Correlated Stochastic Processes.

ABSTRACT

The research dealt with the prediction problem for periodically correlated sequences, that is the stochastic sequences whose mean and covariance structure vary with time in a periodic way. We aimed at sequences with large periods. It has been known already for years that in order to do a reliable forecasting of periodically correlated sequences with large period (or continuous time processes) the standard method of rephrasing the problems in terms of multivariate stationary sequences does not work because of a huge number of unknown parameters. Our main effort was to develop an alternative technique for analysis such sequences . In the first published paper we proposed a new method based on a notion of a "square factor" of the spectrum of the process. In subsequent two papers we showed that this technique is very efficient. We successfully used it to study structure, regularity, autoregressive representation, innovation, and other questions related to prediction of periodically correlated sequences.

Enter List of papers submitted or published that acknowledge ARO support from the start of the project to the date of this printing. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

Received		Paper
05/12/2015	2.00	Andrzej Makagon, Abolghassem Miamee. Spectral Representation of Periodically CorrelatedSequences, Probability and Mathematical Statistics, (03 2013): 175. doi:
TOTAL:		**1**

Number of Papers published in peer-reviewed journals:

(b) Papers published in non-peer-reviewed journals (N/A for none)

Received	Paper
TOTAL:	

Number of Papers published in non peer-reviewed journals:

(c) Presentations

1. Nineteen Annual Research Symposium, Hampton University, April 2014 (poster),
2. AMS Annual Meeting, Baltimore, 16-18 January, 2014,
3. 13th Conference on Probability, Bedlewo, Poland, 18 - 23 May, 2014,
4. XXXII International Seminar on Stability Problems for Stochastic Models, NUST, Trondheim, Norway, 16-21 June, 2014.

Number of Presentations: 0.00

Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received Paper

TOTAL:

Number of Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received Paper

TOTAL:

Number of Peer-Reviewed Conference Proceeding publications (other than abstracts):

(d) Manuscripts

Received		Paper
05/12/2015	3.00	Andrzej Makagon. Innovation and Factorization of the Density of a Regular PC Sequence., Probability and Mathematical Statistics (05 2014)

TOTAL: 1

Number of Manuscripts:

Books

Received	Book

TOTAL:

Received		Book Chapter
05/12/2015	1.00	Andrzej Makagon, Abolghassem Miamee. Structure of PC Sequences and the 3rd Prediction Problem, in: Cyclostationarity: Theory and Methods, Switzerland: Lecture Notes in Mechanical Engineering, Springer 2014, (09 2014)

TOTAL: **1**

Patents Submitted

Patents Awarded

Awards

Graduate Students

NAME	PERCENT_SUPPORTED	Discipline
Trent Miller	1.00	
FTE Equivalent:	**1.00**	
Total Number:	**1**	

Names of Post Doctorates

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Names of Faculty Supported

NAME	PERCENT_SUPPORTED	National Academy Member
Andrzej Makagon	0.25	
Abolghassem Miamee	0.25	
FTE Equivalent:	**0.50**	
Total Number:	**2**	

Names of Under Graduate students supported

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Student Metrics

This section only applies to graduating undergraduates supported by this agreement in this reporting period

The number of undergraduates funded by this agreement who graduated during this period: 0.00

The number of undergraduates funded by this agreement who graduated during this period with a degree in science, mathematics, engineering, or technology fields:...... 0.00

The number of undergraduates funded by your agreement who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields:...... 0.00

Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale):...... 0.00

Number of graduating undergraduates funded by a DoD funded Center of Excellence grant for Education, Research and Engineering:...... 0.00

The number of undergraduates funded by your agreement who graduated during this period and intend to work for the Department of Defense 0.00

The number of undergraduates funded by your agreement who graduated during this period and will receive scholarships or fellowships for further studies in science, mathematics, engineering or technology fields:...... 0.00

Names of Personnel receiving masters degrees

NAME
Trent Miller
Total Number: 1

Names of personnel receiving PHDs

NAME
Total Number:

Names of other research staff

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Sub Contractors (DD882)

Inventions (DD882)

Scientific Progress

See Attachment

Technology Transfer

Proposal No. 58387-MA-H

Prediction Theory of Periodically Correlated Stochastic Processes

Final Progress Report 02/09/2011- 08/15/2014

Abolghassem Miamee
Department of Mathematics
Hampton University, Hampton, VA 23668

The research addressed problems related to prediction of periodically correlated sequences, that is the stochastic sequences whose mean and covariance structure vary with time in a periodic way. We aimed at sequences with large periods. If the period, say T, is small, a periodically correlated sequence can be viewed as components of a T-variate stationary sequence and many problems can be successfully solved using multivariate stationary techniques, e.g. VARMA modeling. If the period is large (or time is continuous) this technique fails because a huge number of unknown parameters produce substantial computational difficulties and estimation errors.

It has been known already for years that in order to do a reliable forecasting of periodically correlated sequences with large period (or continuous time processes) a new methodology, not based on multivariate stationary sequences, has to be used. Our main effort was to develop theoretical foundations for this new approach, and we have been quite successful in doing this. At the end of the project (in Spring 2014), with a help of a graduate student who was supported by the grant, we attempted to create computer algorithms which could be used for modeling and forecasting periodically correlated signals having large period. This part is still in progress.

Our main achievements and the contribution to science was to develop a new method of theoretical analysis of periodically correlated sequences based on a notion of a "square factor" of the spectrum of the process.

The existence a "square factor" of the spectrum of a periodically correlated sequence was first time proved in the supported by the grant paper Makagon, A., and Miamee, A.G., "Spectral Representation of Periodically Correlated Sequences" that appeared in Probability Math. Stat. 33 (1), pp. 175 – 188, (2013). In the case of stationary sequence, square factors of the spectral density of the process play a fundamental role in the prediction theory of stationary sequences. The spectrum of a periodically correlated sequence with period T > 1 consists of T measures and, until publication of the mentioned above paper, there was no construction of a simultaneous square factor for a finite set of measures. In the paper we showed that such construction is possible, namely we proved that there exists a measure ν and a square integrable vector function f such that each of the T components μ_k of the spectrum of a periodically correlated sequence can be represented by this function and its translations as follows

$$\mu_k(\Delta) = \left(\frac{1}{T}\right)\int_\Delta f(t)f\left(t+\frac{2\pi k}{T}\right)^* \nu(dt)$$

The theorem yields a periodically correlated analogue of Kolmogorov's isomorphism theorem which forms a base of the theory of stationary sequences. The whole paper is devoted to the proofs of these two facts.

The theorem itself is not trivial and it turns out that it has deep connections with the group representation theory and so called canonical commutation relation, which stems from quantum mechanics. These connections allowed us to describe a structure of a periodically correlated sequence as an interplay of two unitary operators U,V satisfying canonical commutation relation that act on one vector. This description was included in the paper Makagon, A., and Miamee, A.G., "Structure of PC Sequences and the 3rd Prediction Problem." in: Cyclostationarity: Theory and Methods, Eds. F. Chaari, J. Leskow, A. Neapolitano, A. Sanchez-Ramirea, Lecture Notes in Mechanical Engineering, Springer 2014, 53 - 72. Also in that paper we studied an impact of our factorization and structure theorems on prediction problems including characterization of regularity of the sequence, problem of a positivity of the angle between past and future, related to this question of autoregressive representation of a predictor, and the interpolation problem.

To complete the theoretical part of the project we needed to relate square factors of the spectrum to the innovation coefficients of the sequence, as it was done for stationary sequences by Wiener et.al. Roughly speaking innovations are normalized prediction errors and it is known that every regular sequence can be written as a series of innovation vectors with some coefficients. Finding these coefficients is considered to be a solution to the theoretical prediction problem. The paper Makagon, A., (2014) "Innovation and Factorization of the Density of a Regular PC Sequence.", which will appear in Probability Math. Stat., is devoted to that question. In the paper we found an explicit relation between innovation coefficients and Fourier coefficients of square factors of the spectrum, and described the optimal factor in terms of subspaces of a vector Hardy space H^2.

Although seems very theoretical this last result is important from the point of view of applications. An estimation technique for spectral densities of a periodically correlated sequence based on a periodogram is already well known and carried out by some statistical software. However, so far there was not known how to use periodogram to estimate the innovation coefficients. The results of the paper mentioned above may lead to developing such procedures, and this is what we are planning to do in the future.

We want to thank Army Research Office for supporting this research,

With best regards

Abolghassem Miamee (PI)
Andrzej Makagon (Co-PI)